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Magnetic Field Generation from Alfven Waves Propagating along Helical Lines of Force

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A static induced magnetization of Alfven waves (IMAW) due to the inverse Faraday effect is defined and studied. The Alfven waves are assumed to propagate along helical lines of force of a force-free ambient magnetic field. This induced magnetization follows from the magnetic moment of ordered gyrating motion of charges in the presence of electromagnetic waves and an ambient field. The helicity of the force-free field is found to decrease due to this IMAW. This effect is expected to be important in the physics of magnetization of the sun and pulsars, and also in laboratory devices for the generation of plasmas and their heating.

1. INTRODUCTION

In this paper we consider magnetic field generation from Alfven waves, which occur as the low-frequency limit of electromagnetic (EM) waves, propagating parallel to the direction of a static magnetic field in a two-component plasma. The magnetic field, including a static part, is generated by the inverse Faraday effect (IFE), which is the collective effect of magnetic moments of the ordered motion of charges, in the presence of EM waves, in materials including plasmas. Here Alfven waves are assumed to propagate along helical magnetic lines of force in cold magnetized plasmas. Cylindrical coordinates and the linearized approximation of the equation of motion have been used, ignoring the self-field. So, our results are expected to be valid for applied oscillating electric fields near the reflection region. In plasmas this

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effect is a consequence of wave–particle interaction. One suitable model of plasma for this purpose is that of a collection of free mobile charges of several species. Another model, of a single material medium with non-negligible material pressure and continuity aspects and some assumptions of phenomenological macroscopic MHD behavior, seems to be not so suitable. The electric and magnetic fields **E** and **H** are therefore microscopic in nature and not the polarized fields of the MHD theory.

In general, the IFE is that of the magnetic moment of the ordered loop motion of charges due to EM waves and occurs along the locus of the centers of curvature of these motions, which is also the path of rays of the waves. In the simplest cases, considered earlier (Steiger and Woods, 1972; Chakraborty *et al.*, 1990), it is due to the ordered circular motion of charges in the presence of circularly polarized waves of rectilinear propagation.

For strong, high-frequency EM waves, the induced magnetization of the IFE develops mainly from the electron current, because the ion current is not a dominating factor for fields of moderate or weak intensity, and is not significantly affected by the ambient field. But, for Alfven waves, which are of low frequencies, the situation is not so, because then, simultaneously, both electron and ion motions have to be taken into account.

The ambient magnetic field is assumed to be that of force-free field configurations. In the equilibrium state the electrostatic force is negligible, and the pressure gradient exactly balances the Lorentz force. With the help of this balance condition we study the dependence of pressure on the net static magnetic field.

Magnetization in the megagauss range generated by fast electrons in the tail of the distribution has been detected with high-power lasers and discussed (Steiger and Woods, 1972; Briand *et al.*, 1985; Chakraborty *et al.*, 1988). Estimation of the Faraday rotation effect of backscattered radiation for the field has been reported (Briand *et al.*, 1985). A magnetic field is known to exist from turbulent generation in electrically conducting materials and from thermal generation in laser-produced plasmas. Another source of dc magnetic field is the IFE. For strong, high-frequency EM waves the IFE has been theoretically formulated (Stenflo, 1977) and experimentally demonstrated in solids (Pershan, 1963) and in plasmas (Steiger and Woods, 1972; Chakraborty *et al.*, 1990; Vanderziel *et al.*, 1965).

The IMAW is expected to be significant in the physics of the sun and others stars, including pulsars. It is also relevant in radiofrequency heating of tokamaks and other laboratory plasmas. Alfven waves are supposed to be one of the main heating agencies of the solar corona. The surface Alfven waves deposit energy in a resonance region and several such regions together provide the heat required to raise the solar corona temperature to millions of degrees.

Here we assume the existence of a force-free magnetic field, given by the solution of the equation

$$\nabla \times \mathbf{H} = \alpha \mathbf{H} \tag{1}$$

where α is a constant along a magnetic field line. Equation (1) is satisfied by some geometries of two-dimensional field lines only. It has no solution for any one-dimensional field. So, the results of Chakraborty *et al.* (1990) cannot be obtained as a special case of the present work. The field reversal suggested by them is found to be an erroneous conclusion, as the ratio of the total field to the ambient field there is always positive.

The lines of force of the ambient magnetic field, as well as the direction of the wave ray path, therefore make up a system of right circular helices. The assumption of a force-free ambient field is valid in many astrophysical problems. The solution of equation (1) for the force-free ambient field **H**, in cylindrical polar coordinates (r, θ, z), is given by

$$\mathbf{H} = H_0(0, J_1(\alpha r_0), J_0(\alpha r_0)) \tag{2}$$

H_0 is the field amplitude and r_0 is the unperturbed value of the position vector **r**.

Since the vector potential of **H** is $\mathbf{A} = \alpha^{-1}\mathbf{H}$, the helicity density, given by

$$\mathbf{A} \cdot \mathbf{H} = \alpha^{-1}\mathbf{H}^2 = \alpha^{-1} H_0^2\{[J_1(\alpha r_0)]^2 + [J_0(\alpha r_0)]^2\} \tag{3}$$

is uniform. We assume that $\alpha r_0 \ll 1$. So, approximately, the ambient field is

$$\mathbf{H} = H_0(0, \tfrac{1}{2}\alpha r_0, 1) \tag{4}$$

Since $\alpha r_0 \ll 1$ and $H_z \gg H_\theta$, the helicity is very weak.

The induced IFE field is present only in the θ direction and is zero in the $\hat{r}$ and $\hat{z}$ directions. It is diamagnetic in nature, and it effectively reduces the magnetic field in the θ direction. Hence, H_z becomes still larger compared to H_θ due to the IFE field, and the helicity is reduced thereby.

2. FORMULATION OF THE PROBLEM

Any charged particle of the species s, gyrating with velocity $\mathbf{u}_s$, has the magnetic dipole moment

$$\boldsymbol{\mu}_s = \frac{1}{2c}(\mathbf{R}_s \times \mathbf{j}_s) \tag{5}$$

where $\mathbf{R}_s$ is the position vector of the charge q_s with respect to a fixed origin, $\mathbf{j}_s$ $(= N_s q_s \mathbf{u}_s)$ is the surface density of the current, and N_s is the number density. The corresponding induced magnetic field, denoted by $\mathbf{H}_i$, is given by $\mathbf{H}_i = \sum_s 4\pi N_s \boldsymbol{\mu}_s$. In a neutral plasma, since $N_e = N_i = N_0$ (say), we find that

$$\mathbf{H}_i = 4\pi N_0(\boldsymbol{\mu}_e + \boldsymbol{\mu}_i) \tag{6}$$

where $\boldsymbol{\mu}_e$ is the magnetic dipole moment due to the motion of an electron and $\boldsymbol{\mu}_i$ is that due to ionic motion.

3. MAGNETIZATION OF OSCILLATING ELECTRIC FIELD IN CYLINDRICAL GEOMETRY

The equation of motion of an electron in an EM field is

$$\dot{\mathbf{U}}_e = \left(-\frac{e}{m}\right)\mathbf{E} - \left(\frac{e}{mc}\right)(\mathbf{U}_e \times \mathbf{H}) \tag{7}$$

where $\mathbf{U}_e$ is the velocity and m is the mass of an electron, and $\mathbf{E}$ is the electric intensity vector; Ω_e $(= eH/mc)$ is the gyration frequency. Similarly, the equation of ion motion is

$$\dot{\mathbf{U}}_i = \left(\frac{e}{M}\right)\mathbf{E} + \frac{e}{Mc}(\mathbf{U}_i \times \mathbf{H}) \tag{8}$$

where M is mass of the ion and $\mathbf{U}_i$ is the ionic velocity.

The expression for the ambient magnetic field is

$$\mathbf{H} = \left(0, \frac{H_0}{2}\alpha r_0, H_0\right) \tag{9}$$

The applied electric field may be expressed as

$$E_r = a\, e^{i\varphi}, \qquad E_\theta = ia\, e^{i\varphi}, \qquad E_z = 0 \tag{10}$$

where $\phi = kz - \omega t$, with k the wave number and ω the wave frequency, and a is the amplitude of the applied field.

To solve equations (7) and (8), we write

$$r_s = r_0^s + r^s, \qquad \theta_s = \theta_0^s + \theta^s, \qquad z_s = z_0^s + z^s$$

Here $(r_0^s, \theta_0^s, z_0^s)$ represents the initial position of the particle, (r^s, θ^s, z^s) gives the corresponding perturbations, and (r_s, θ_s, z_s) is the instantaneous position coordinates. The solution of equation (7) is

$$\mathrm{Re}(r^e) = A_e \cos\varphi, \qquad \mathrm{Re}(\theta^e) = B_e \sin\varphi, \qquad \mathrm{Re}(z^e) = C_e \cos\varphi \tag{11}$$

where Re stands for the real part, with

$$A_e = \frac{ea}{m\omega} \frac{\omega + \Omega_{ez}}{\omega^2 - \Omega_{ez}^2 - \Omega_{e\theta}^2}$$

$$B_e = -\frac{ea}{m\omega^2 r_0^e} \frac{\omega^2 - \Omega_{e\theta}^2 - \omega\Omega_{ez}}{\omega^2 - \Omega_{ez}^2 - \Omega_{e\theta}^2} \tag{12}$$

$$C_e = \frac{ea\Omega_{e\theta}}{m\omega^2} \frac{\omega + \Omega_{ez}}{\omega^2 - \Omega_{e\theta}^2 - \Omega_{ez}^2}$$

$$\Omega_{ez} = \frac{eH_z}{mc}, \qquad \Omega_{e\theta} = \frac{eH_\theta}{mc} \tag{13}$$

The solution of equation (8) is expressed as

$$\mathrm{Re}(r^i) = A_i \cos\varphi, \qquad \mathrm{Re}(\theta^i) = B_i \sin\varphi, \qquad \mathrm{Re}(z^i) = C_i \sin\varphi \tag{14}$$

where

$$A_i = -\frac{ea}{M\omega} \frac{\omega - \Omega_{iz}}{\omega^2 - \Omega_{i\theta}^2 - \Omega_{iz}^2}$$

$$B_i = \frac{ea}{M\omega^2 r_0^i} \frac{\omega^2 - \Omega_{i\theta}^2 - \omega\Omega_{iz}}{\omega^2 - \Omega_{i\theta}^2 - \Omega_{iz}^2} \tag{15}$$

$$C_i = \frac{ea\Omega_{i\theta}}{M\omega^2} \frac{\omega - \Omega_{iz}}{\omega^2 - \Omega_{i\theta}^2 - \Omega_{iz}^2}$$

$$\Omega_{iz} = \frac{eH_z}{Mc}, \qquad \Omega_{i\theta} = \frac{eH_\theta}{Mc} \tag{16}$$

The components of the induced magnetization, evaluated from equation (6) and the field solutions (11)–(16), are

$$\langle H_{ir} \rangle = 0 \tag{17}$$

$$\langle H_{i\theta} \rangle = \frac{2\pi Ne\omega}{c} (A_i C_i - A_e C_e) \tag{18}$$

$$\langle H_{iz} \rangle = 0 \tag{19}$$

where $\langle P \rangle$ $[= (1/T) \int_0^T P\, dT]$ is the average of the quantity P over the time period T $(= 2\pi/\omega)$.

4. THE ALFVEN WAVE APPROXIMATION $|\Omega| \gg |\omega|$

Now using these values of A's and B's and the condition $m \ll M$ in equations (17)–(19), we find that

$$\langle H_{ir} \rangle = 0 \tag{20}$$

$$\langle H_{i\theta} \rangle = -4\omega_{pe}^2 \frac{a^2}{\omega^2 H_0} \frac{\alpha r_0}{(4+\alpha^2 r_0^2)^2} \tag{21}$$

$$\langle H_{iz} \rangle = 0 \tag{22}$$

where the plasma frequency is $\omega_{pe} = (4\pi N e^2/m)^{1/2}$ and $r_0 = r_0^e = r_0^i$.

Since $\alpha r_0 \ll 1$, $\alpha^2 r_0^2$ is neglected with respect to 4, so that

$$\langle H_{i\theta} \rangle = -4\omega_{pe}^2 \frac{a^2}{\omega^2 H_0} \frac{\alpha r_0}{16}$$

$$= -4k_1 \alpha r_0 \tag{23}$$

where k_1 is a constant. The graph of $\langle H_{i\theta} \rangle$ vs. (αr_0) is plotted in Figure 1.

5. PRESSURE VARIATION WITH αr_0

When, in the equilibrium state, the pressure gradient is balanced by the Lorentz force, we have the relation

$$-\nabla p + \frac{1}{4\pi} [\nabla \times (\mathbf{H} + \mathbf{H}_i) \times (\mathbf{H} + \mathbf{H}_i)] = 0 \tag{24}$$

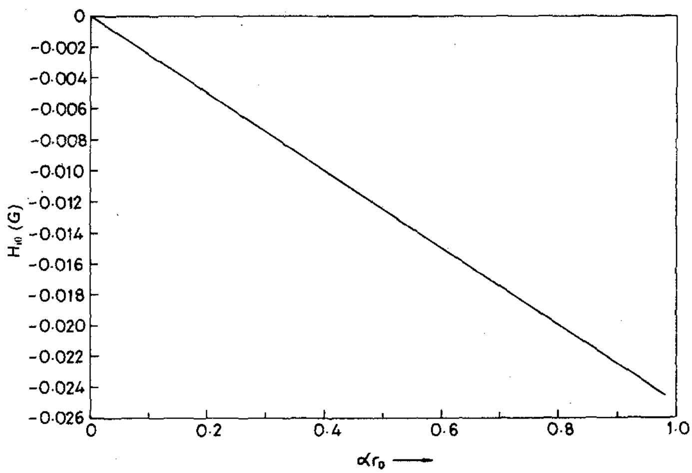

Fig. 1. Nature of the variation of $\langle H_{i\theta} \rangle$ with αr_0 for an astrophysical plasma.

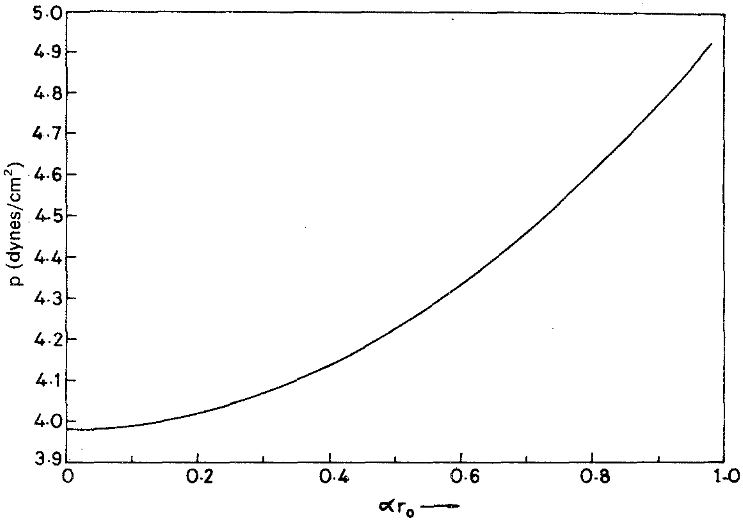

Fig. 2. Variation of pressure with αr_0 for an astrophysical plasma.

Hence

$$p=\frac{1}{8\pi}[\langle H_z\rangle^2+(\langle H_\theta\rangle+\langle H_{i\theta}\rangle)^2]$$

Using the values of H_z, H_θ, and $H_{i\theta}$ from (4) and (23), respectively, we find that

$$p=\frac{1}{8\pi}\left[H_0^2+(\alpha r_0)^2\left(\frac{H_0}{2}-4k_1\right)^2\right] \tag{25}$$

The graph of p vs. αr_0 is plotted in Figure 2.

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